INTEGRALI PER SOSTITUZIONE $\boxed{V\emptyset}$

① $\int (\underbrace{2x+1}_{t})^7 dx =$ $\qquad t = 2x+1$

$dt = 2\,dx \Rightarrow dx = \frac{1}{2}dt$

$= \int t^7 \cdot \frac{1}{2}dt = \frac{1}{2}\int t^7 dt = \frac{1}{2}\frac{t^8}{8} + c = \frac{1}{16}t^8 + c = \frac{1}{16}(2x+1)^8 + c$

② $\int 3x\,(\underbrace{1-x^2}_{t})^3 dx$ $\qquad t = 1-x^2$

$dt = -2x\,dx \Rightarrow x\,dx = -\frac{1}{2}dt$

$= 3\int t^3\left(-\frac{1}{2}dt\right) = -\frac{3}{2}\int t^3 dt = -\frac{3}{2}\frac{t^4}{4} + c = -\frac{3}{8}t^4 + c$

$= -\frac{3}{8}(1-x^2)^4 + c$

③ $\int \frac{\sqrt{\ln x}}{x}dx =$ $\qquad t = \ln x$

$dt = \frac{1}{x}dx$

$= \int \underbrace{\sqrt{\ln x}}_{t} \cdot \underbrace{\frac{1}{x}dx}_{dt} = \int \sqrt{t}\,dt = \int t^{\frac{1}{2}}dt = \frac{t^{\left(\frac{1}{2}+1\right)}}{\frac{3}{2}} + c =$ $\qquad \frac{1}{2}+1 = \frac{3}{2}$

$= \frac{2}{3}\sqrt{t^3} + c = \frac{2}{3}\sqrt{\ln^3 x} + c$

④ $\int \frac{x}{1+x^2}dx =$ $\qquad t = 1+x^2$

$dt = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}dt$

$= \int \frac{1}{\underbrace{1+x^2}_{t}} \cdot \underbrace{x\,dx}_{\frac{1}{2}dt} = \int \frac{1}{t}dt = \ln|t| + c = \ln|1+x^2| + c$

$= \ln(1+x^2) + c$

⑤ $\int \cos x\,\operatorname{sen}^3 x\,dx =$ $\qquad t = \operatorname{sen} x$

$dt = \cos x\,dx$

$= \int t^3 dt = \frac{t^4}{4} + c = \frac{\operatorname{sen}^4 x}{4} + c$

⑥ $\int x\cos(1-x^2)\,dx$ $\qquad t = 1-x^2$

$dt = -2x\,dx \Rightarrow x\,dx = -\frac{1}{2}dt$

$= \int \cos t \cdot \left(-\frac{1}{2}dt\right) = -\frac{1}{2}\int \cos t\,dt = -\frac{1}{2}\operatorname{sen} t + c = -\frac{1}{2}\operatorname{sen}(1-x^2) + c$

⑦ $\int x^2 e^{x^3+1}dx$ $\qquad t = x^3+1$

$dt = 3x^2dx \Rightarrow x^2dx = \frac{1}{3}dt$

$= \int e^t\left(\frac{1}{3}dt\right) = \frac{1}{3}\int e^t dt = \frac{1}{3}e^t + c = \frac{1}{3}e^{x^3+1} + c$

(8) $\int e^{-x} dx$

$t = -x$

$dt = -dx \Rightarrow dx = -dt$

$= \int e^{t} (-dt) = -\int e^{t} dt = -e^{t} + c = -e^{-x} + c$